Human-centered Theorem Proving: Interactive and automated

Wilfried Sieg[1],

[1] Carnegie Mellon University

Hilbert asked in 1900 for an investigation of proofs through his 24th Problem (that was discovered only recently); in 1918, he called for a "theory of the specifically mathematical proof". Gentzen insisted in 1936, "the objects of proof theory shall be the proofs carried out in mathematics proper"; he viewed derivations in his *natural deduction calculi* (nd calculi) as "formal images [Abbilder]" of such proofs. Clearly, these formal images were to represent significant structural features of arguments from standard mathematical practice. Gentzen's conception of formalizing mathematical proofs yields a mapping from proofs to nd derivations, but it is static and neglects the dynamics of proof construction. Using *normal intercalation calculi* (nic calculi) one can insist that formal proofs reflect both the structure *and* the construction of mathematical proofs. I will formulate the nic calculus for classical first-order logic and sketch (1) the proof of its completeness, (2) the interactive verification of the Cantor-Bernstein Theorem in ZF set theory, and (3) the automated search for Gödel's proofs when taking for granted as axioms representability and derivability conditions. The search mechanism exploits the fact that nic calculi are "bi-directional" and "goal-directed"; they generate partial derivations or derivations with gaps. This feature allows the formulation of strategies involving forward and backward steps that narrow the gap between assumptions and goals. The mechanism has been implemented in the theorem prover AProS and yields a sophisticated computational model of reasoning in logic and elementary set theory. In turn, it underpins a dynamic intelligent *proof tutor* that is used in a web-based introduction to logic, *Logic & Proofs*, and supports students who are learning the strategically guided construction of proofs.